Example: Graph the following inequality

\[ X > 1 \]
ex) Solve the following and graph the solution

\[ x + 3 < -5 \]
Ex) Solve the following and graph the solution

\[ Y + 3 \geq 10 \]
Ex) Solve the following and graph the solution

\[ 5a \leq 25 \]
Ex. Solve and graph the solution set on a number line:

\[ 4x - 3 > -23 \]

\[ 4x - 3 > -23 \]

\[ 4x > -20 \]

\[ x > -5 \]

The solution set is \((-5, \infty)\).
Ex. Solve and graph the solution set on a number line:

\[
\frac{x - 4}{2} \geq \frac{x - 2}{3} + \frac{5}{6}
\]

\[
\frac{x - 4}{2} \geq \frac{x - 2}{3} + \frac{5}{6}
\]

\[
6\left(\frac{x - 4}{2}\right) \geq 6\left(\frac{x - 2}{3} + \frac{5}{6}\right)
\]

\[
3(x - 4) \geq 2(x - 2) + 5
\]

\[
3x - 12 \geq 2x - 4 + 5
\]

\[
3x - 12 \geq 2x + 1
\]

\[
x \geq 13
\]

The solution set is \([13, \infty)\).
Ex) Translate the following phrase into an equivalent inequality statement

X is greater than -2 and at most 4

-2 < x < 4
Ex) Translate the following phrase into an equivalent inequality statement

X is less than -4 or at least 1

\[ x < -4 \text{ or } x \geq 1 \]
Compound Inequality with and

\[ 2x \geq x - 1 \quad \text{and} \quad 3x \geq 3 + 2x \]

\[ 2x \geq x - 1 \]

\[ x \geq -1 \]

\[ 3x \geq 3 + 2x \]

\[ x \geq 3 \]

x \geq -1 \quad \text{and} \quad x \geq 3
Compound Inequality with and

\[ x + 3 < 1 \text{ and } x - 4 > -12 \]

\[ x + 3 < 1 \quad x - 4 > -12 \]

\[ x < -2 \quad x > -8 \]

\[ x < -2 \text{ and } x > -8 \]
Homework) Graph the solution sets for the following compound inequalities. Then write each solution set using interval notation

$3x + 2 < -3$ or $3x + 2 > 3$
Homework) Solve, graph, and find the interval notation of your solution

\[ x + 3 < 1 \text{ and } x - 4 > -12 \]
The intersection of sets $A$ and $B$, written $A \cap B$, is the set of elements common to both set $A$ and set $B$. This definition can be expressed in set-builder notation as follows:

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$$
EXAMPLE
Find the intersection.

(a) \{1,3,5,7\} \cap \{2,4,6,8,10\}  \quad (b) \{1,3,7\} \cap \{2,3,8\}

SOLUTION

(a) \{1,3,5,7\} \cap \{2,4,6,8,10\}

Because the two sets have nothing in common, there is no solution. Therefore, we say the solution is the empty set: \(\emptyset\).

(b) \{1,3,7\} \cap \{2,3,8\}

Both sets have the element 3. That is the only element they have in common. Therefore, the solution set is \{3\}. 
Ex. Find the intersection: \( \{3, 4, 5, 6, 7\} \cap \{3, 7, 8, 9\} \)

Elements 3 and 7 are common to both sets.

\( \{3, 4, 5, 6, 7\} \cap \{3, 7, 8, 9\} = \{3, 7\} \)
The union of sets $A$ and $B$, written $A \cup B$, is the set of elements that are members of set $A$ or set $B$ or of both sets. This definition can be expressed in set-builder notation as follows:

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}.$$
EXAMPLE

Find the union of the sets.

\[ \{1,3,7,8\} \cup \{2,3,8\} \]

SOLUTION

The solution will be each element of the first set and each element of the second set as well. However, we will not represent any element more than once. Namely the elements 3 and 8 should not be listed twice.

\[ \{1,2,3,7,8\} \]
Ex. Find the union: \( \{3,4,5,6,7\} \cup \{3,7,8,9\} \)

The union is the set consisting of all the elements from each set.

\( \{3,4,5,6,7\} \cup \{3,7,8,9\} = \{3,4,5,6,7,8,9\} \)
EXAMPLE

Solve the compound inequality. Use graphs to show the solution set to each of the two given inequalities, as well as a third graph that shows the solution set of the compound inequality.

\[ x - 4 \leq 2 \quad \text{and} \quad 3x + 1 > -8 \]

SOLUTION

1) Solve each inequality separately. We wish to isolate \( x \) in each inequality.

\[ x - 4 \leq 2 \]
\[ x \leq 6 \quad \text{Add 4 to both sides} \]
Intersection & Compound Inequalities

CONTINUED

\[3x + 1 > -8\]
\[3x > -9\]
\[x > -3\]

Subtract 1 from both sides

Divide both sides by 3

Now we can rewrite the original compound inequality as:

\[x \leq 6 \text{ and } x > -3\]
2) Take the intersection of the solution sets of the two inequalities. Now we can solve each half of the compound inequality.

\[ x \leq 6 \]

\[ x > -3 \]

The parenthesis stays in position.

\[ x \leq 6 \text{ and } x > -3 \]

The bracket stays in position.

Therefore the solution is \((-3, 6]\).