5.6 A GENERAL REVIEW: FACTORING
Recall:

- Factor – a # that “goes into” another # without remainder
  Ex: Factors of 20: 1, 2, 4, 5, 10, 20

- Factoring - “undoing” multiplication (division)
Methods of Factoring

- The Greatest Common Factor (any # of terms)
- Factoring by Grouping (4 terms)
- Factoring Trinomials:
  - Guess and Check (3 terms)
  - AC Method (3 terms)
- Difference of Sums (two terms)
- Sum and Difference of Cubes (two terms)
Greatest Common Factor

- Greatest common factor – the largest monomial that is a factor of each term
  - largest piece that is common to all terms
  - must have that much to pull out of each term

Ex:

1. $5x^3 - 15x$
2. $30xy^2 - 25x^2y$
3. $36x^5 + 72x^3 - 81x^2$
Factoring by Grouping

- Expression usually has 4 terms
- Factor out GCF if possible
- Factor using groups (1\textsuperscript{st} two terms, 2\textsuperscript{nd} two)

Ex: \(xy + 2x + 4y + 8\)

\[x(y + 2) + 4(y+2)\]

- Notice: Both expression in ( ) are the same.
- Factor out common expression \((y+2)\); left with another expression

\[(y+2)(x+4)\]
Examples

1. $6x^2 - 3x - 4x + 2$

2. $3ax + 21x - a - 7$

3. $8x^3 - 12x^2 + 14x - 21$
Things to think about

- When both terms have a negative, a negative is “common” to both so pull it out.
- If one term is negative and one is positive, look at the first factored expression and see if you need to factor out a negative or leave it.
- Always check by reapplying the distributive property to make sure it is correct.
- You never have to miss this type of problem if you take the time to check it.
Factoring Trinomials

- **Goal:** To split the trinomial into 2 pieces that multiply to give us what we started with.
- **Always factor out GCF first, if possible.**

- **Form:** $ax^2 + bx + c$ when $a = 1$
  
  $ax^2 + bx + c = (x + p)(x + q)$
  
  - Product of $p \times q = c$
  - Sum of $p + q = b$

- **Note:** The sum could be a sum or difference depending on the sign of the $c$, or last, term.
\( ax^2 + bx + c \)

1. If C term:
   - positive – Add; both signs are the same which means we add the 2 factors to get b or the middle term
   - negative – Subtract; one sign is positive, one is negative which means the signs are different so we subtract the 2 factors to get b or the middle term

2. The B term sign tells us:
   - what the 2 signs of the factors are if they are the same (if negative both negative; if positive both positive)
   - What the larger sign of the factors is if they are different (one is positive, one is negative)
Example

\[ x^2 - 7x + 10 \]

1. Do we have a GCF?
2. Split the 1\(^{st}\) term into 2 equal parts
   \[ (x \quad ) (x \quad ) \]
3. C term (3\(^{rd}\) term) is positive, therefore we have the same signs which implies adding (so both are negative or both positive)
4. B term is negative, so both are negative
   \[ (x - \quad ) (x - \quad ) \]
5. What 2 numbers add to give 7 and multiply to give 10? (You may want to list the factors of 10, to make this easier)
6. \( (x - 5) (x - 2) \) The order does not matter.
Examples
Reminder: Check to see if you have a GCF first.
1.  \( x^2 + 7x + 6 \)

2.  \( a^2 - 7a + 12 \)

3.  \( x^2 - 4x - 5 \)

4.  \( y^2 + y - 42 \)

5.  \( 3a^2 - 3a - 60 \)
Perfect Square Trinomial

- First and last term are perfect squares
- Last term is always positive
- Middle term = 
  \[2 \times (\text{square root of } a) \times (\text{square root of } b)\]
- Signs of factors are dependent on middle term

Example:

\[x^2 - 6x + 9\]

- a & c terms are perfect squares
- last term is positive.

\[(x - 3)(x - 3)\]
AC Method

- Form: $ax^2 + bx + c$ when $a \neq 1$

1. Factor out GCF, if possible
2. Rewrite trinomial as a polynomial with 4 terms
   - Multiply coefficients $a$ & $c$ in $ax^2 + bx + c$; $P = ac$
   - Split this product into 2 pieces that will combine to give the $bx$ term; $S/D = b$
3. Factor by grouping
4. Check by multiplying
Example

\[2x^2 + 5x + 3\]

Ex: \[2x^2 + 5x + 3\] Leading term ≠ 1

1. \[P = ac = 2 \times 3 = 6\]

2. \[S = 5\] (sum not diff. since last sign is +)

What 2 factors of 6 multiply to give 6 and add to give 5? (Both #'s are positive since 1\textsuperscript{st} sign is +)

\[2x^2 + 5x + 3\]

\[2x^2 + 2x + 3x + 3\] (just rewriting the b term)

Now factor by grouping.

\[2x(x + 1) + 3(x + 1)\]

\[(2x + 3)(x + 1)\]
Examples

1. $3y^2 + 14y - 5$
2. $20x^2 + 9x - 20$
3. $6x^2 - 51x + 63$
4. $2x^4 - 34x^3 + 64x^2$
The difference of Squares

- Two terms (binomial)
- Both perfect squares
- Minus sign (difference) separating two terms
- Does not work if addition sign (sum)
- Split into 2 factors taking the square root of each: one positive, one negative
- Always factor out GCF first

Example:

\[ x^2 - 25 = (x + 5)(x - 5) \]
Examples

1. $a^2 - 64$
2. $9x^2 - 25$
3. $4a^2 + 25$
4. $2x^2 - 18$
5. $a^4 - 16$
Sum and difference of Cubes

- Two terms
- Both are perfect cubes
- Formulas (SOAP)

\[ a^3 + b^3 = (a+b)(a^2 - ab + b^2) \]
\[ a^3 - b^3 = (a-b)(a^2 + ab + b^2) \]

Example:

\[ x^3 + 27 = (x + 3) (x^2-3x+9) \]

since \[ a = x \ & \ b = 3 \] (take the cube root)
Examples

1. $a^3 - 8$
2. $t^3 + 125h^3$
3. $10a^3 - 640b^3$
4. $(1/8)x^3 - (1/27)y^3$
Things to look for:

1. Always factor out GCF
2. Look at the # of terms
   1. Two: Difference of cubes? Squares?
   2. Three: AC Method, Perfect Square Trinomial
   3. Four: Factor by grouping
3. Can it be factored further? Repeat steps
4. Check your answer by multiplying
Examples

1. \( x^2 - 18x + 81 \)
2. \( 15x^2 + 11x - 6 \)
3. \( 21y^2 - 25y - 4 \)
4. \( x^3 - x^2 \)
5. \( 3y^2 - 9y - 30 \)
6. \( x^2 - 64 \)